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Rigid body rotation and block internal discretization in DDA analysis

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SUMMARY

In the original formulation of DDA by Shi, a linear displacement function term is used. This has the limitations of uncontrolled block and stress distortion due to rigid body rotation. In the present paper, the authors propose a new iterative method which can avoid the distortion due to the rotation even when the rotation or number of time-step is large. Furthermore, the authors propose a simple internal discretization scheme which is applicable for both concave and convex polygon which is particularly important for a large block. The stress and strain distribution with a large block can be obtained with ease under this scheme. The numerical examples as shown have demonstrated the advantages of the present proposal in DDA analysis.

INTRODUCTION

Discontinuous Deformation Analysis (DDA) and Manifold Method[1] which is an energy-based method is an alternative to Distinct Element Method for discontinuity based problems. Under this method, artificial damping is not required which is clearly a definite advantage over the Distinct Element Method by Cundall. There are however many drawbacks to the original form of DDA and the authors have resolved some of these limitations.[2] In the present paper, the authors address another two major drawbacks of DDA which are block and stress distortion due to rotation and poor representation of strain and stress with a large block.

In the original DDA formulation, a linear displacement function term is used by Shi.[3] Under this formulation, angular rotation will induce strain term which may be very large if the angular rotation within a time-step is "large". However, rigid body rotation should not induce strain or distortion of block, and this phenomenon is particularly important if the time-step is relatively large. The authors have the experience of block expansion by more than 50 per cent due to rigid body rotation alone. Another error is the rotation of the local reference frame and hence distortion of the stress and strain within a block. It may however be difficult to determine beforehand the amount of angular rotation within a time-step for a general problem unless a

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trial- and-error process is performed. If a small time-step is used to control the amount of rotation, it will be very time consuming to perform the DDA analysis which is a computationally demanding process. MacLaughlin and Sitar[4] have proposed to use the second-order term to account for the effects of angular rotation. This will be effective for an angular rotation of 0·4 or less within a time-step. Ke proposed to use a post-adjustment with a maximum allowable rotation of 0·1 radian within a step.[5] Even a small time-step is used, the error within each time-step of these methods however will still accumulate so that the distortion of block may still be noticeable after several hundred time-steps. In the present paper, the authors propose an iterative method which can reduce the block distortion to a minimum. Actually, the iteration is performed in the main iteration analysis within each step so that no additional iterative analysis will be required. The present approach will only require minor extra computation and the increase in solution time is nearly negligible.

The second limitation of the original DDA formulation is also related to the linear displacement function used for the blocks. The stress and strain within each block are constant which are basically only the average values and this is definitely a poor representation for a large block. Koo has proposed to use a third-order displacement function but this may still be inadequate for a large block. In the present paper, the authors propose to use a simple internal discretization scheme which can maintain a low aspect ratio but is applicable to both concave and convex block to obtain the stress and strain distribution within each large block. The internal sub-blocks within each large block are tied by four stiff springs along each common boundary so as to maintain the integrity of each block.

ITERATIVE ANALYSIS FOR RIGID BODY ROTATION

Consider a block with centroid (x_0, y_0), centroidal translation (u_0, y_0), centroidal rotation (θ) and strain $(\varepsilon_x, \varepsilon_y, \gamma_{xy})$, the exact solution for the x- an y-components (u, v) of the displacement of an arbitrary point (x, y) within a deformable block are given by

$$u = u_0 + (x - x_0)(\cos\theta - 1) - (y - y_0)\sin\theta + (x - x_0)\varepsilon_x + (y - y_0)\varepsilon_{xy}/2 \tag{1}$$

$$v = v_0 + (x - x_0)\sin\theta + (y - y_0)(\cos\theta - 1) + (y - y_0)\varepsilon_y + (x - x_0)\varepsilon_{xy}/2 \tag{2}$$

Shi's linear displacement function terms are equivalent to assume θ to be small so that $\sin\theta \approx \theta$ and $\cos\theta \approx 1$. Equations (1) and (2) hence reduce to

$$u = u_0 - (y - y_0)\theta + (x - x_0)\varepsilon_x + (y - y_0)\varepsilon_{xy}/2 \tag{3}$$

$$v = v_0 + (x - x_0)\theta + (y - y_0)\varepsilon_y + (x - x_0)\varepsilon_{xy}/2 \tag{4}$$

Equations (3) and (4) are actually adopted by Shi in his DDA formulation and form the basis of the DDA method. MacLaughlin and Sitar have pointed out that the representation of $\sin\theta$ by θ has an error term of $O(\theta^3)$. They view that such approximation is good enough for general use. On the other hand, the error term in representing $\cos\theta$ as 1 has an error term of $O(\theta^2)$. They hence proposed to use $1 - \theta^2/2$ to represent the term $\cos\theta$ in order to increase the accuracy of the displacement term. In general, this is a satisfactory way which is applicable to many problems. The authors however note that the errors accumulated after hundreds of time-steps may be noticeable as will be demonstrated later.

The authors propose to use an iterative approach to further increase the accuracy of the computation with only minor increase in the computation. Consider that

$$\cos\theta - 1 = -\sin^2\theta/(1+\cos\theta) \tag{5}$$

Put equation (5) in equations (1) and (2), we have

$$u = u_0 + \left[-(x-x_0)\frac{\sin\theta}{1+\cos\theta} - (y-y_0)\right]\sin\theta + (x-x_0)\varepsilon_x + (y-y_0)\varepsilon_{xy}/2 \tag{6}$$

$$v = v_0 + \left[-(y-y_0)\frac{\sin\theta}{1+\cos\theta} + (x-x_0)\right]\sin\theta + (y-y_0)\varepsilon_y + (x-x_0)\varepsilon_{xy}/2 \tag{7}$$

The two terms in the square brackets in equations (6) and (7) are now denoted by f_1 and f_2 for simplicity. If we take f_1 and f_2 to be constants, then equations (6) and (7) are still linear functions of $\sin\theta$ term. Under this condition, the derivation of the stiffness sub-matrix terms by Shi can all be retained on the condition that θ is replaced by $\sin\theta$.

The block displacement function $[T_i]$ is now given by

$$\begin{bmatrix} 1 & 0 & -(x-x_0)\dfrac{\sin\theta}{1+\cos\theta} - (y-y_0) & x-x_0 & 0 & (y-y_0)/2 \\ 0 & 1 & -(y-y_0)\dfrac{\sin\theta}{1+\cos\theta} + (x-x_0) & 0 & y-y_0 & (x-x_0)/2 \end{bmatrix} \tag{8}$$

and the displacement vector $[D_i]$ is given by

$$[D_i] = \{u_0 \;\; v_0 \;\; \sin\theta \;\; \varepsilon_x \;\; \varepsilon_y \;\; \gamma_{xy}\}^{\mathrm{T}} \tag{9}$$

In the treatment of the terms f_i and f_2 in iteration step i within each time-step, the value of θ can be taken as the value of θ in step $i-1$. This process goes along with the global iteration analysis within each time-step. The error introduced in this formulation come from the difference between θ_{i-1} and θ_i. If there is no angular acceleration or equivalently if the block undergoes a rigid body rotation, the present approach will give an exact representation of the strain term which is better than that by MacLaughlin and Sitar. When angular acceleration is present, the errors introduced in the present formulation is usually small unless a very large time-step is used. Furthermore, MacLaughlin and Sitar[4] have not considered the situation of angular acceleration in their formulation of the various terms which is also a major drawback. The numerical example in the latter section of this paper will clearly demonstrate the advantage of the present formulation.

MODIFICATIONS OF LINE LOAD AND VISCOSITY FORCE TERMS

In the present formulation, the block transformation matrix has been modified. For the various stiffness and force terms associated with DDA, only those which are affected by $[T_i]$ are required

to be modified. The first one is the application of line load to a block. If a line load with a length L is applied inside a block from point i (x_i, y_i) to point j (x_j, y_j) with a uniform load of (F_x, F_y) with $F_x = F_x(t)$; $F_y = F_y(t)$; $0 \leqslant t \leqslant 1$, the associated force term is given by Shi[3]

$$\int_0^1 [T_i]^{\mathrm{T}} \begin{pmatrix} F_x \\ F_y \end{pmatrix} L\,\mathrm{d}t \to [F_i] \tag{10}$$

since

$$\int_0^1 (x - x_0)\,L\,\mathrm{d}t = \frac{L}{2}(x_2 + y_1 - 2x_0) \tag{11}$$

$$\int_0^1 (y - y_0)\,L\,\mathrm{d}t = \frac{L}{2}(y_2 + y_1 - 2y_0) \tag{12}$$

The force term $[F_i]$ is hence given by

$$L \begin{bmatrix} F_x \\ F_y \\ -\tfrac{1}{2}(y_2 + y_1 - 2y_0)\left(F_x + \dfrac{\sin\theta}{1+\cos\theta}F_y\right) - \tfrac{1}{2}(x_2 + x_1 - 2x_0)\left(\dfrac{\sin\theta}{1+\cos\theta}F_x - F_y\right) \\ \tfrac{1}{2}(x_2 + x_1 - 2x_0)F_x \\ \tfrac{1}{2}(y_2 + y_1 - 2y_0)F_y \\ \tfrac{1}{4}(y_2 + y_1 - 2y_0)F_x + \tfrac{1}{4}(x_2 + x_1 - 2x_0)F_y \end{bmatrix} \to [F_i] \tag{13}$$

Referring to the force of viscosity, the stiffness and force terms as given by Shi[4] are

$$\frac{2M}{\Delta^2} \int\!\!\int [T_i]^{\mathrm{T}} [T_i]\,\mathrm{d}x\,\mathrm{d}y \to [k_{ii}] \tag{14}$$

$$\frac{2M}{\Delta} \left(\int\!\!\int [T_i]^{\mathrm{T}} [T_i]\,\mathrm{d}x\,\mathrm{d}y \right) [V_0] \to [F_i] \tag{15}$$

where $[V_0]$ is the initial velocity of a block at the start of a time-step. To determine the integral terms in equations (14) and (15) in the iteration analysis,

$$\text{let } \bar{x} = x - x_0, \quad \bar{y} = y - y_0 \tag{16}$$

$$[T_i]^{\mathrm{T}}[T_j] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -\bar{y} - \bar{x}\dfrac{\sin\theta}{1+\cos\theta} & \bar{x} - \bar{y}\dfrac{\sin\theta}{1+\cos\theta} \\ \bar{x} & 0 \\ 0 & \bar{y} \\ \bar{y}/2 & \bar{x}/2 \end{bmatrix} \begin{bmatrix} 1 & 0 & -\bar{y} - \bar{x}\dfrac{\sin\theta}{1+\cos\theta} & x & 0 & \bar{y}/2 \\ 0 & 1 & \bar{x} - \bar{y}\dfrac{\sin\theta}{1+\cos\theta} & 0 & \bar{y} & \bar{x}/2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -\bar{y} - \bar{x}\dfrac{\sin\theta}{1+\cos\theta} & \bar{x} & 0 & \bar{y}/2 \\ 0 & 1 & \bar{x} - \bar{y}\dfrac{\sin\theta}{1+\cos\theta} & 0 & \bar{y} & \bar{x}/2 \\ -\bar{y} - \bar{x}\dfrac{\sin\theta}{1+\cos\theta} & \bar{x} - \bar{y}\dfrac{\sin\theta}{1+\cos\theta} & \left(-\bar{y} - \bar{x}\dfrac{\sin\theta}{1+\cos\theta}\right)^2 + \left(\bar{x} - \bar{y}\dfrac{\sin\theta}{1+\cos\theta}\right)^2 & -\overline{xy} - \bar{x}^2\dfrac{\sin\theta}{1+\cos\theta} & \overline{xy} - \bar{y}^2\dfrac{\sin\theta}{1+\cos\theta} & -\dfrac{\bar{y}^2}{2} + \dfrac{\bar{x}^2}{2} - \overline{xy}\dfrac{\sin\theta}{1+\cos\theta} \\ \bar{x} & 0 & -\overline{xy} - \bar{x}^2\dfrac{\sin\theta}{1+\cos\theta} & \bar{x}^2 & 0 & \overline{xy}/2 \\ 0 & \bar{y} & \overline{xy} - \bar{y}^2\dfrac{\sin\theta}{1+\cos\theta} & 0 & \bar{y}^2 & \overline{xy}/2 \\ \bar{y}/2 & \bar{x}/2 & \bar{x}/2 - \bar{y}^2/2 - \overline{xy}\dfrac{\sin\theta}{1+\cos\theta} & \overline{xy}/2 & \overline{xy}/2 & \bar{x}^2/4 + \bar{y}^2/4 \end{bmatrix} \tag{17}$$

The integral term $\iint [T_i]^{\mathrm{T}}[T_i]\,\mathrm{d}x\,\mathrm{d}y$ is equal to

$$\begin{bmatrix} s & 0 & 0 & 0 & 0 & 0 \\ 0 & s & 0 & 0 & 0 & 0 \\ 0 & 0 & (s_1+s_2)\dfrac{2}{1+\cos\theta} & -s_3-s_1\dfrac{\sin\theta}{1+\cos\theta} & s_3-s_2\dfrac{\sin\theta}{1+\cos\theta} & \dfrac{(s_1-s_2)}{2}-s_3\dfrac{\sin\theta}{1+\cos\theta} \\ 0 & 0 & -s_3-s_1\dfrac{\sin\theta}{1+\cos\theta} & s_1 & 0 & s_3/2 \\ 0 & 0 & s_3-s_2\dfrac{\sin\theta}{1+\cos\theta} & 0 & s_2 & s_3/2 \\ 0 & 0 & \dfrac{(s_1-s_2)}{2}-s_3\dfrac{\sin\theta}{1+\cos\theta} & s_3/2 & s_3/2 & (s_1+s_2)/4 \end{bmatrix} \tag{18}$$

where s is the area of the block, $s_1 = \iint (x^2 - x_0 x)\,\mathrm{d}x\,\mathrm{d}y$, $s_2 = \iint (y^2 - y_0 y)\,\mathrm{d}x\,\mathrm{d}y$, $s_3 = \iint (xy - x_0 y)\,\mathrm{d}x\,\mathrm{d}y$.

INTERNAL DISCRETIZATION OF BLOCK

In the classical DDA formulation, the stress and strain obtained within any block are constant. These constant values are basically the average values of the actual stress and strain within a block. If the size of a block is small, this approach may be acceptable. For a large block where the variation of stress and strain within the block may be significant, this approach is obviously unacceptable. Koo[6] has proposed to use a third-order displacement function to increase the accuracy of stress and strain determination within a block. This approach is suitable for many problems but may still be inadequate when the size of the block is very large or the variation of stress and strain very rapid. For the general cases, the authors view that this limitation can be overcome by: (1) finite element internal discretization of a block; (2) internal discretization by constant stress/strain DDA block with the introduction of internal springs to ensure continuity within each block. For the first approach, it will be troublesome to implement because finite element and DDA are based on totally different concept and the resulting algorithm and program will be very complicated. For ease of programming, the authors have adopted the second approach.

The authors have adopted the second approach by using an automatic internal discretization scheme which generates triangular sub-blocks within a block. The use of an automatic block discretization is very important as it can greatly relieve the burden of the user in performing DDA analysis. With reference to Figure 1, the local node number of the block is numbered in a counterclockwise manner as 1, 2, 3, 4, 5. To start with the discretization, the user must prescribe the largest edge size a_0 of the sub-block. The number of subdivision within each side of a block is obtained by taking the smallest integer N_i such that $l_i/N_i \leqslant a_0$, where l_i is the length of the side of

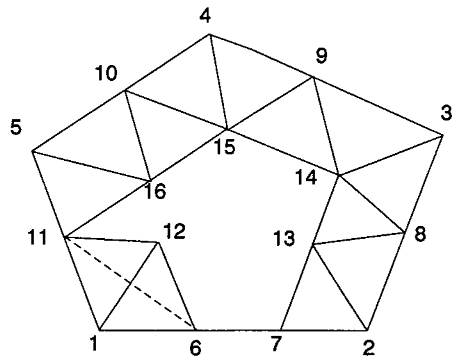

Figure 1. Internal discretization of a block.

a block. The subdivided nodes are then numbered consecutively in a counterclockwise manner as shown in Figure 1.

The second step in the internal discretization is to cycle from points 1 to 11 in a counterclockwise manner as 1, 6, 7, 2, 8, 3, 9, 4, 10, 5, 11. Consider a typical point 1 with its neighbouring points 6 and 11. The distance between 6 and 11 is denoted by d. This procedures for sub-block formation are:

1. If d is less than $\sqrt{2}a_0$ or the angle between points 11-1-6 is less than 120°, points 6 and 11 are joined. It should be noted that d greater than $\sqrt{2}a_0$ is not necessarily equivalent to angle 11-1-6 greater than 90° because the length of the block is most commonly less than a_0. If the resulting line is within the polygon of the block, this line and the corresponding sub-block are accepted and the procedure proceeds to point 2. If the resulting line lies outside the polygon of the block (or the block is concave at this region), this line is rejected and the sub-block is not formed. Point 2 will then be considered. In Figure 1, sub-block 11-1-6 is hence not valid and will not be formed.
2. If d is greater than $\sqrt{2}a_0$ or the angle between points 11-1-6 is greater than 120°, a bisector of angle 11-1-6 as shown in Figure 2 is drawn and the length of the bisector is set to a_0. It should be noted that it is not really necessary to form this bisector for an angle less than 120° in actual practice. This angle can also be changed very easily in a computer program so that the user can select an angle which he thinks is suitable. Points 6–12 and 11 and 12 are then joined with the formation of two sub-blocks and point 1 will no longer be on the list of sub-block division. Point 12 will take the place of point 1 in the sub-division procedure. For the problem as shown in Figure 1, the points under consideration after the first round of sub-division are 12, 6, 7, 13, 14, 15, 16, 11.
3. The results from step 2 is checked for the following cases:
   (i) If the number of vertices is greater than 4, repeat steps 1 and 2.
   (ii) If the number of vertices is equal to 4, the quadrilateral is divided into two triangles with the condition that the shorter diagonal is connected in order to keep the aspect ratio low.
   (iii) If the number of vertices is equal to 3, the job is completed.

This block-subdivision scheme is easy to implement and is suitable for concave and convex blocks. A graphics file in the form of DXF format generated from the program developed by the

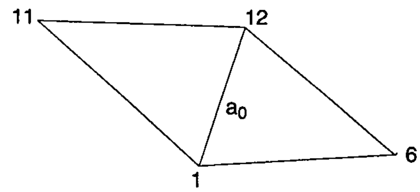

Figure 2. Bisection in sub-blocks formation.

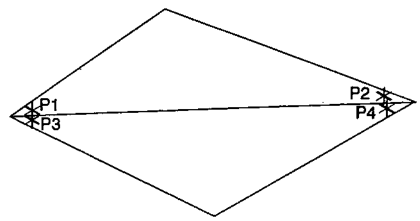

Figure 3. Application of internal springs.

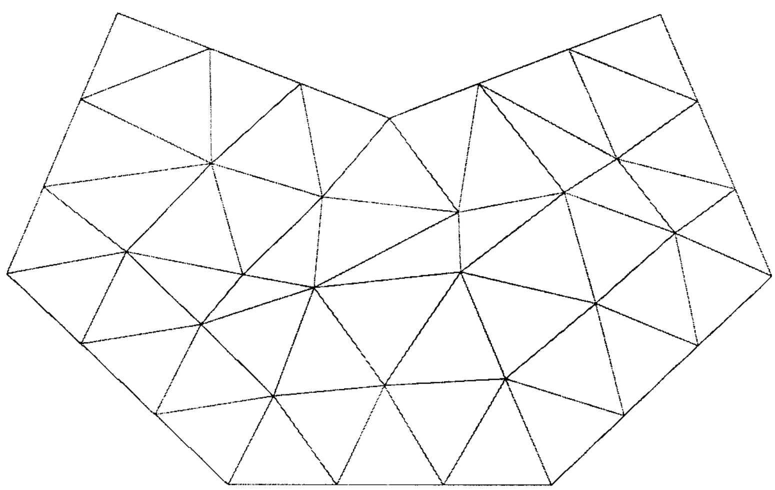

Figure 4. Internal discretization within a concave block.

authors is shown in Figure 4 which can illustrate the applicability of the present scheme to concave block. A special feature of the present scheme is that the aspect ratio is kept small under all steps so as to increase the accuracy of DDA analysis. After the sub-blocks generation, the sub-blocks must be tied at the common sides so as to maintain compatibility (continuity) with a block. The sub-blocks are tied by four springs between P1–P3, P2–P4, P1–P4 and P2–P3 (Figure 4). The separation between the sub-blocks are then effectively restrained and the sub-blocks can be treated as classical blocks by DDA. The introduction of these internal springs to maintain continuity as adopted by the authors are actually similar to the penalty as used by

Lin.[7] The present scheme is chosen because spring is available in the programs developed by the authors and such implementation will be easy to implement automatically.

NUMERICAL EXAMPLES AND DISCUSSION OF RESULTS

To demonstrate the applicability of the previous improvement to DDA, two numerical examples will be considered here. The first example is a rock fall problem where both angular acceleration and rigid block rotation will be involved. It is an important topic in Hong Kong and other countries and DDA is a good method for such problem. Consider the rock fall problem as shown in Figure 5(a). The parameters used for the present analysis are:

time step $= 0{\cdot}02$ s, friction angle between blocks $= 20^{\circ}$, Poisson's ratio $= 0{\cdot}34$, density $= 2\times10^{4}$ kg/m^3, $E = 1\times10^{10}$ Pa, $K_{\mathrm{N}} = 3\times10^{6}$ N/m, $K_{\mathrm{s}} = 1\times19^{6}$ N/m.

In the present analysis, the joint is modelled with tangential and normal springs[2] (with stiffness k_{N} and k_{s}) as proposed by the author instead of the penalty used by Shi.[4] A typical side of the block with a length 0·5 m is taken in the study and the results are studied as follows:

case 1 = original DDA formulation by Shi
case 2 = method by MacLaughlin
case 3 = method by authors

From Table I, it is clear that the use of second-order term by MacLaughlin has largely eliminated the error due to rigid body rotation. The errors may however still be noticeable if they accumulate through a number of time-steps. The method proposed by the authors is however very accurate even when the number of time-steps is large. Under the proposed method, extra time is required to compute the more complicated matrices as given before. No extra iteration will be required for the proposed method because the iteration as described before goes with the main iteration

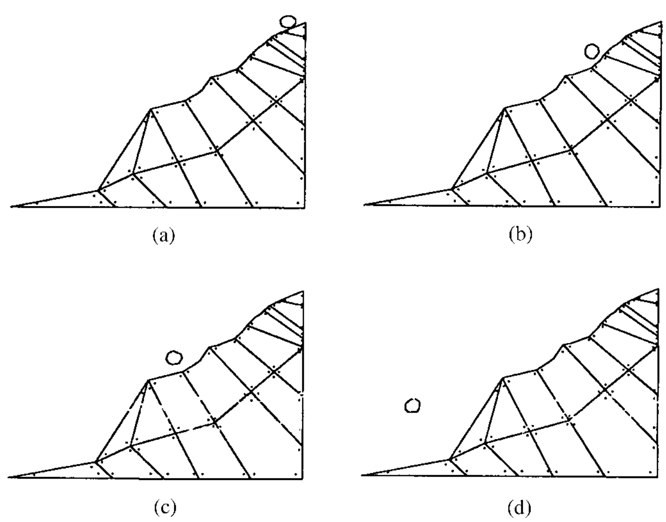

Figure 5. Rock fall problem: (a) step 0; (b) step 80; (c) step 130; (d) step 180.

Table I. Comparison of results from original DDA formulation, MacLaughlin formulation and the formulation by the authors

	Results at time-step 80		Results at time-step 130		Results at time-step 180	
	Length (m)	Error (%)	Length (m)	Error (%)	Length (m)	Error (%)
Case 1	0·613105	22·6211	0·659972	31·9943	0·676955	35·3911
Case 2	0·500162	0·0324	0·502881	0·5762	0·503796	0·7592
Case 3	0·5000005	0·0001	0·5000015	0·0003	0·500002	0·0004

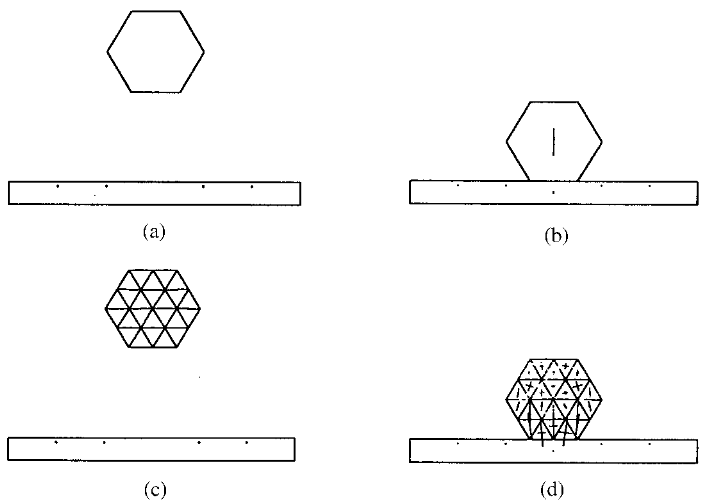

Figure 6. Free fall and impact of a hexagon: (a)–(d).

analysis of the DDA analysis. The extra time used for the proposed method is found to be negligible as much of the time is used for the solution of equation and iterative analysis. The author's formulation is hence a very effective method in dealing with rigid body rotation associated with DDA analysis.

The second numerical example is the impact of a hexagon onto a flat surface which is shown in Figure 6. If the whole hexagon is a deformable block, the stress and strain distribution within the hexagon will basically be the average values within the block which is far from satisfactory. Within the internal discretization scheme proposed by the authors, the stress and strain distribution within the block can be obtained. This is particularly important if the block may fracture into smaller pieces when the induced stresses is great enough. The present scheme may hence be employed to model the formation of new cracks and blocks and the process of block fracture which is important in many problems. In the present example, the side of the hexagon has a length of 1 m and is allowed to fall free from a height of 2 m. The parameters used for the analysis are:

$E = 1 \times 10^8$ Pa, Poisson ratio $= 0{\cdot}34$, $\phi = 0^\circ$, $K_N = 3 \times 10^6$ N/m, $K_s = 1 \times 10^6$ N/m, time step $= 0{\cdot}1$ s.

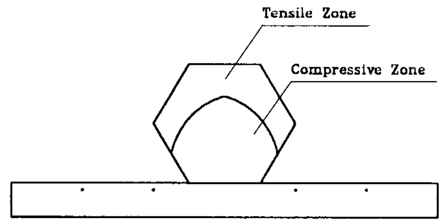

Figure 7. Tensile compressive zone for stress YY.

A calculation based on Newtonian mechanics gives a prediction of impact at a time-step of 6·4. From the numerical computation, impact is found to occur within time-step 7 which agrees with classical mechanics. Since the stress and strain distribution after the impact within block varies with time-step, the comparison of results are taken at a time-step of 60 under which the stresses are at the maximum. Figure 6(a) represents the block at time-step 0 and Figure 6(b) shows the principal stresses at a time-step of 60. The stresses in Figure 6(b) are $\sigma_x = -34$ Pa and $\sigma_y = -18\,225$ Pa ($-$ ve means compressive stress). In Figure 6(c), the same block is discretized into 24 sub-blocks with an edge length of 0·5 m. The principal stresses vectors are shown in Figure 6(d) at a time-step of 60. It is very clear that the results of Figure 6(d) differs considerably from that as shown in Figure 6(b) and it is a definite advantage of internal discretization of a large block. Since σ_y is the major principal stress, it has to be examined in greater details. As shown in Figure 7, while the majority of the block is under compression, tensile stress is actually induced at the upper part of the block. The results obtained may be used to check whether the block will fail in compression or fracture under tensile stress. The results of Figures 6(d) and (7) has clearly illustrated the importance of internal discretization in DDA analysis. Without refined internal stress and strain distribution within a block, many problems cannot be studied properly with DDA.

CONCLUSION

DDA is a powerful tool in the analysis of discontinuous medium. However, DDA is at present less powerful than the well-developed Distinct Element Method since it has many limitations. The authors have addressed and overcome some of these limitations of the previous works and have proposed an iterative method to account for the rigid body rotation and an internal discretization scheme to model a large block in the present paper. From the numerical results for the rigid body rotation study, it is found that the proposed scheme is a very accurate scheme even when the number of time step is large. The proposed method also has the advantage of requiring negligible increase in computation time.

The most important improvement to DDA is however the introduction of an internal discretization scheme which depends only on one input: the maximum edge distance of a sub-block. Under the proposed scheme, the aspect ratio can be kept low so as to maintain a good shape of sub-block and increase the accuracy of the computation. The detailed stress and strain distribution within a block can be obtained and this is very important in many real problems. Actually, without such refined analysis, DDA cannot be a powerful tool to the analysis of discontinuous medium.

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